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LRS Bianchi Type-I Models in Self-Creation Cosmology

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Abstract: *We have investigated the field equations of Barber's second self-creation theory with perfect fluid source for an LRS Bianchi type-I metric by using variable deceleration parameter. The Barber's field equations of self-creation theory have been Solved for two cases. The first case involving a power law solution describes the dynamics of universe from big-bang to present epoch while the second case admit an exponential solution seems reasonable to project dynamics of future universe. It has been found that Barber's scalar function contributes a very high matter density with highest pressure at early stage of the universe. Other geometrical and physical aspects of the models are also discussed.*

Keywords: *Bianchi type-I model, Self-creation theory, deceleration parameter, scalar function.*

1.Introduction

Barber [1] has proposed two continuous self-creation theories by modifying the Brans-Dicke theory [2] and Einstein's general theory of relativity. These modified theories create the universe out of self-contained gravitational and matter fields. Brans-Dicke [2] theory develops Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution of matter in motion. However, Barber [1] has included continuous creation of matter in these theories. His first theory is a modification of the Brans-Dicke [2] theory and the second theory is a modification of Einstein's general theory of relativity. Brans [3] has pointed out that Barber's first theory is not only in disagreement with experiment, but is actually inconsistent. The second theory of Barber is a modification of general relativity to a variable G theory and predicts local effects within the observational limits. In this theory the scalar fields do not directly gravitate, but simply divide the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar field couples with the trace of the energy-momentum tensor. In view of the consistency of Barber's second self-creation theory of gravitation many authors [4-10] investigated various aspects of different space-times. Venkateswarlu and Pawan Kumar [11] have studied the role of higher dimensional FRW models in Barber's second self-creation theory when the source of gravitation is a perfect fluid. Pradhan and Vishwakarma [12] have studied LRS Bianchi type-I cosmological models in Barber's second self creation theory. Tiwari and Jyotsna [13] investigated anisotropic Bianchi type-I model in self creation cosmology. Recently, Katore et al. [14] have studied Bianchi type-I dark energy cosmological model with polytropic equation of state in Barber's second self-creation cosmology.

Motivated by the above investigations, in this paper, we have obtained cosmological solutions for an LRS Bianchi type-I metric in Barber's second self-creation theory of gravitation with perfect fluid for variable deceleration parameter models for the universe. This paper is organized as follows: The metric and field equations are presented in Section 2. In Section 3, we deal with the general solutions of the field equations. Power expansion model and Exponential expansion model are presented in Subsections 3.1 and 3.2 respectively. Concluding remarks are given in Section 4.

2.The Metric and Field Equations :

We consider an LRS Bianchi type-I metric in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2\left(dy^2 + dz^2\right), \qquad (1)$$

where $A = A(x,t), B = B(x,t)$. The field equations in Barber's second self-creation theory [1] are

$$R_{ij} - \frac{1}{2} R g_{ij} = -\frac{8\pi T_{ij}}{\phi} \qquad (2)$$

and

$$\Box\, \phi = \phi_{;k}^{;k} = \frac{8}{3}\pi\lambda T\,, \qquad (3)$$

where λ is a coupling constant to be determined from the experiments, ϕ is a Barber's scalar function and T is the trace of energy-momentum tensor.

The energy-momentum tensor T_{ij} for a perfect fluid distribution is given by

$$T_{ij} = (p+\rho)u_i\, u_j - p g_{ij} \qquad (4)$$

with

$$g_{ij}\, u^i\, u^j = 1, \tag{5}$$

where u_i is the four-velocity of the fluid and p and ρ are the proper isotropic pressure and energy density, respectively.

The Barber's field Equations (2) and (3), for the matric (1) with the help of Equations (4) and (5), take the form :

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{B'^2}{A^2B^2}=-\frac{8\pi p}{\phi}, \tag{6}$$

$$\dot{B}'-\frac{B'\dot{A}}{A}=0, \tag{7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{B''}{A^2B}+\frac{A'B'}{A^3B}=\frac{-8\pi p}{\phi}, \tag{8}$$

$$\frac{2B''}{A^2B}-\frac{2A'B'}{A^3B}+\frac{B'^2}{A^2B^2}-\frac{2\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}=\frac{8\pi\rho}{\phi}, \tag{9}$$

$$\ddot{\phi}+\frac{\dot{A}\dot{\phi}}{A}+\frac{2\dot{B}\dot{\phi}}{B}+\frac{A'\phi'}{A^3}-\frac{2B'\phi'}{A^2B}-\frac{\phi''}{A}=(\rho-3p)\frac{8\pi\lambda}{3} \tag{10}$$

Here the overhead dash and dot denote partial differentiation with respect to x and t respectively.

3. Solutions of the Field Equations

Equation (7), after integration, yields

$$A=\frac{B'}{\mu}, \tag{11}$$

where μ is an arbitrary function of *x*.

Equations (6) and (8), with the help of (11) reduce to

$$\frac{B}{B'}\frac{d}{dx}\left(\frac{\ddot{B}}{B}\right)+\frac{\dot{B}}{B'}\frac{d}{dt}\left(\frac{B'}{B}\right)+\frac{\mu^2}{B^2}\left(1-\frac{B\mu'}{B'\mu}\right)=0. \tag{12}$$

If we assume $\frac{B'}{B}$ to be a function of *x* alone, then A and B are separable in *x* and *t*. Hence, Equation (12) yields on integration

$$B=\mu R(t), \tag{13}$$

where $R(t)$ is an arbitrary function of *t*.

Using (13) in Equation (11), we obtain

$$A = \frac{\mu'}{\mu} R(t). \tag{14}$$

With the help of the aforesaid metric potentials obtained in (13) and (14) and a suitable transformation the metric (1) becomes

$$ds^2 = dt^2 - R^2(t)\left[dX^2 + e^{2X}(dy^2 + dz^2)\right], \tag{15}$$

where $X = \ln \mu$.

Using (13) and (14) in Equations (6), (9) and (10), we obtain

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{1}{R^2} = -\frac{8\pi p}{\phi}, \tag{16}$$

$$\frac{3}{R^2} - \frac{3\dot{R}^2}{R^2} = \frac{8\pi\rho}{\phi}, \tag{17}$$

$$\ddot{\phi} + \frac{3\dot{\phi}\dot{R}}{R} - \frac{3\phi'\mu}{\mu' R^2} - \frac{\mu^2}{\mu'^2 R^2}\frac{d}{dx}\left(\frac{\phi'}{\mu'}\right) = (\rho - 3p)\frac{8\pi\lambda}{3}. \tag{18}$$

For the sake of simplicity, if we assume ϕ to be a function of *t* only, then Equation (18) with the help of Equation (16) and (17), reduces to

$$\frac{\ddot{\phi}}{\phi} + \frac{3\dot{\phi}\dot{R}}{\phi R} - \frac{2\lambda\ddot{R}}{R} = 0. \tag{19}$$

The function *R*(*t*) remains undetermined. To obtain its explicit dependence on t, one may have to introduce additional assumptions. To achieve this, we assume the deceleration parameter to be variable i.e.

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b \text{ (variable)}, \tag{20}$$

where $H = \dot{R}/R$ is the Hubble parameter. The above Equation can be written as

$$\frac{\ddot{R}}{\dot{R}} + b\frac{\dot{R}}{R} = 0. \tag{21}$$

The general solution of Equation (21) is given by

$$\int e^{\int \frac{b}{R}dR}\, dR = kt + l, \tag{22}$$

where *k* and *l* are constants of integration.

In order to solve the problem completely, we have to choose $\int \frac{b}{R} dR$ in such a manner that Equation (22) becomes integrable.

Without loss of generality, we assume

$$\int \frac{b}{R} dR = \ln\ L(R), \tag{23}$$

which does not effect the nature of generality of solution.

By using (23) in Equation (22), we get

$$\int L(R) dR = kt + l. \tag{24}$$

Of course the choice of L(R), in Equation (24), is quite arbitrary but, since we are looking for physically viable models of the universe consistent with observations, we consider the following cases.

3.1 Power Expansion Model

Let us consider $L(R) = k_1 R^{n-1}$, where $n \neq 0$ and k_1 is an arbitrary constant.

In this case on integration, Equation (24) gives the exact solution

$$R(t) = (k_2 t + k_3)^{1/n}, \tag{25}$$

where k_2 and k_3 are arbitrary constants.

Hence the metric (15) reduces to the form

$$ds^2 = dt^2 - (k_2 t + k_3)^{2/n} \left[dX^2 + e^{2X}(dy^2 + dz^2)\right]. \tag{26}$$

In this case, by using (25), Equation (19) reduces to

$$(k_2 t + k_3)^2 \ddot{\phi} + \frac{3k_2}{n}(k_2 t + k_3)\dot{\phi} - \frac{2(1-n)\lambda k_2^2}{n^2}\phi = 0, \tag{27}$$

which on integration yields

$$\phi = e^{\frac{-(3-n)t}{2n}} \left[c_1 e^{\sqrt{m_1}\ t} + c_2 e^{-\sqrt{m_1}\ t}\right], \tag{28}$$

where

$$m_1 = \frac{(3-n)^2 + 8(1-n)\lambda}{4n^2} \tag{29}$$

and c_1, and c2 are constants of integration.

Using (25) in Equations (16) and (17), we obtain

$$8\pi\ p = \phi\left[\frac{(2n-3)k_2^2 + n^2}{n^2(k_2 t + k_3)^2}\right], \tag{30}$$

$$8\pi\rho = 3\phi\left[\frac{(k^2 - n^2)}{n^2(k_2 t + k_3)^2}\right], \tag{31}$$

where the Barber's scalar function ϕ is given by (28).

The directional Hubble's parameters H_i (i =1, 2, 3), the average Hubble's parameter (H),expansion scalar (θ), shear scalar (σ), spatial volume (V) and the anisotrophy parameter (A_m) of the model (26) are, respectively given by-

$$H_1 = H_2 = H_3 = \frac{\dot{R}}{R} = \frac{k_2}{n(k_2 t + k_3)}, \tag{32}$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{R}}{R} = \frac{k_2}{n(k_2 t + k_3)}, \tag{33}$$

$$\theta = 3H = \frac{3k_2}{n(k_2 t + k_3)}, \tag{34}$$

$$\sigma^2 = \frac{1}{\sqrt{2}}\left[H_1^2 + H_2^2 + H_3^2 - \frac{1}{3}\theta^2\right]^{\frac{1}{2}} = 0, \tag{35}$$

$$V = R^3 = (k_2 t + k_3)^{3/n}, \tag{36}$$

$$A_m = \frac{2\sigma^2}{3H^2} = 0. \tag{37}$$

From the set of solutions obtained in this section, it is easy to see that in this model, at the initial epoch $t \to t_o$, $t_o = -k_3/k_2$, the physical parameters such as p, ρ, θ, H_1, H_2, H_3 and H are all infinite, whereas the volume scalar vanishes. The infinite density and pressure show that the model has a point type singularity at $t = t_o$. From equation (28) it is seen that the Barber's scalar function ϕ is a decreasing function of time when $n < 3$ and increasing function of time when $n \geq 3$, but it will always be positive if both C_1 and C_2 are positive. From equations (30) and (31), it is obvious that both pressure p and energy density ρ are derictly proportional to the Barber's scalar function ϕ and so it's contribution is very high in the values of p and ρ. Therefore the Barber's scalar function ϕ plays a very important role in deciding the physical nature of the universe. The model indicates that as $t \to \infty$, the spatial volume tends to infinity whereas the pressure and energy density tend to zero. It is also observed that shear scalar (σ) and Anisotrophy parameter (A_m) are zero throughout the whole expansion of the universe. Furthermore, $\frac{\sigma}{\theta} = 0$, which confirms the isotrophic nature of the space-time.

3.2 Exponential Expanding Model :

Let $L(R) = \frac{1}{k_4 R}$, where k_4 is an arbitrary constant.

In this case on integration, Equation (24) yields the exact solution.

$$R = m_3 \; e^{m_2 t}, \tag{38}$$

where m_2 and m_3 are arbitrary constants.

Hence, the geometry of the universe, in this case, is described by the line-element.

$$ds^2 = dt^2 - m_3^2 \; e^{2m_2 t} \left[dX^2 + e^{2X} \left(dy^2 + dz^2\right)\right]. \tag{39}$$

In this case, by using (38), Equation (19) takes the form

$$\ddot{\phi} + 3m_2 \, \dot{\phi} - 2\lambda m_2^2 \phi = 0 \,. \tag{40}$$

which on integration gives

$$\phi = e^{\frac{-3m_2 t}{2}} \left[c_3 \; e^{\sqrt{l_2}\, t} + c_4 \; e^{-\sqrt{l_2}\, t}\right], \tag{41}$$

where

$$l_2 = \frac{(9 + 8\lambda) m_2^2}{4}, \tag{42}$$

and c_3, and c_4 are integrating constants.

Using (38) in Equations (16) and (17), we get

$$8\pi p = \phi \left[\frac{1}{m_3^2 \; e^{2m_2 t}} - 3m_2^2\right], \tag{43}$$

$$8\pi \rho = 3\phi \left[\frac{1}{m_3^2 \; e^{2m_2 t}} - m_2^2\right], \tag{44}$$

where ϕ is given by Equation (41).

The directional Hubble's parameters H_i (i = 1, 2, 3), the mean Hubble's parameter (H), expansion scalar (θ), shear scalar (σ), spatial volume (V) and the anisotrophy parameter (A_m) for the model (39) are, respectively given by

$$H_1 = H_2 = H_3 = H = m_2, \tag{45}$$

$$\theta = 3H = 3m_2, \tag{46}$$

$$\sigma = \frac{1}{\sqrt{2}} \left[H_1^2 + H_2^2 + H_3^2 - \frac{1}{3}\theta^2\right]^{\frac{1}{2}} = 0, \tag{47}$$

$$V = R^3 = m_3^3 \; e^{3m_2 t}, \tag{48}$$

$$A_m = \frac{2\sigma^2}{3H^2} = 0\,. \tag{49}$$

From the above set of equations, it is obvious that the model is non-singular since the exponential function never vanishes. Thus this model does not have any physical singularity. For this model the directional Hubble's parameters as well as mean Hubble's parameter are constant and equal to each other. This implies that the universe expands at finite rate in all spatial directions. Constant value of expansion scalar (θ) represents an uniform expansion of the universe. The flow of the fluid is geodesic as the acceleration $f_i = (0, 0, 0, 0)$. In this case also the behaviour of the Barber's scalar function ϕ is quite similar to the previous case. The spatial volume of the universe is finite when $t = 0$ (i.e. initial epoch) and expands exponentially as t increases and becomes infinite when $t = \infty$. The shear scalar and anisotrophy parameter are zero throughout the whole expansion of the universe. Furthermore, $\frac{\sigma}{\theta} = 0$ which implies that our model is isotropic. For $m_2 = 0$, the model reduces to a static radiating model with constant density and pressure.

4.Concluding Remarks

We have presented two categories of exact solutions of Barber's second self-creation theory for an LRS Bianchi type-I model by using variable deceleration parameter. These solutions give two types of cosmological models (i) Power expansion model (ii) Exponential expansion model. The first category of solution corresponds to the singular cosmological model with power-law expansion, whereas the second category of solution corresponds to the non-singular model with exponential expansion. The Barber's scalar function ϕ affects the behaviour of physical parameters. The isotropic pressure p and energy density ρ occupied the highest values at the initial stage of the model due to the presence of the Barber's scalar function ϕ . This shows that the Barber's scalar ϕ contributes a very high density matter with high isotropic pressure in early universe. It is also observed that when $\phi \to 0$, pressure and energy density tend to zero. Furthermore, the ratio $\frac{\sigma}{\theta} = 0$ implies that our both the cosmological models are isotropic which are supported by ΛCDM model. We have also discussed the physical and kinematical properties of the cosmological models.

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